Using Fama-French Model In a Cost-Sharing Agreement

By Stuart Webber


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In this report, Webber explains the Fama-French model and assesses whether that model is appropriate for determining a cost-sharing agreement's discount rate.

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U.S. international tax laws permit multinational enterprises to structure cost-sharing agreements (CSAs) to finance the development of intellectual property and new products.[1] CSAs are instruments to fund IP investments by several legal entities within an MNE. For example, a company might structure an investment so that it is financed by two or more subsidiaries, with one domiciled in the United States and others chartered abroad. The risks and benefits of the CSA are shared between those legal entities within the MNE.

In many situations, a subsidiary contributes more than financial resources to support IP development. It might also contribute preexisting IP to the CSA. Suppose a company originated in the United States, where it funded and developed its IP. If the company later expands abroad and forms international subsidiaries, it might structure a CSA to share IP development costs and risks between several legal entities. Because new IP is likely to build on a company's prior work, the U.S. entity's preexisting IP should be contributed to the CSA. This IP might include patents, trademarks, intellectual know-how, and an experienced workforce. If the overseas entity has no comparable contributions to make to a CSA, it must compensate the U.S.-based entity for the latter's contributions to the CSA. This payment for an IP contribution is commonly called a "buy-in" payment.

To value the buy-in payment, the MNE must project the future earnings generated by the new IP, which is a challenging task. It may be difficult to accurately predict the success of new products or the size of markets, sales, and profit margins. Companies must also discount future profits to calculate an investment's present value. The discount rate has a major impact on the investment's present value and thus the buy-in payment. In an earlier article, I showed an example in which changing the discount rate in a CSA by less than three points reduced the present value of an investment by 23 percent, or $296 million. This reduced the buy-in payment by $178 million.[2] The discount rate in a CSA is an important figure that affects both the worldwide tax rate of an MNE and government tax revenue in the United States and abroad.

In 2009 the IRS and Veritas Software Corp. litigated a CSA buy-in payment. In that case, the IRS and Veritas both used the capital asset pricing model (CAPM) to determine the discount rate in that CSA. Because of CAPM's complexity and differing approaches used to compute a discount rate, the IRS and Veritas calculated different figures. The IRS computed a 13.5 percent discount rate, while Veritas

[1]*See* reg. section 1.482-7 for regulations governing CSAs.

[2]Stuart Webber, "Financial Assumptions Regarding the Sensitivity of IRS Cost-Sharing Regulations," *Tax Notes Int'l*, Nov. 26, 2012, p. 855.

calculated a 20.47 percent rate.[3] Still, both used CAPM to determine the figure. CAPM has also been used to resolve a variety of other legal disputes — including gift and estate tax issues, bankruptcy cases, and IP valuation — so it has several legal precedents.[4] However, IRS regulations do not specify that CAPM must be used to determine a discount rate. In fact, IRS officials have said that they do not intend to provide specific guidance on what method should be used to calculate this figure.[5]

Since CAPM was first developed, financial literature has proposed improvements to the model, and several academicians believe they have developed superior methods to determine discount rates. Because IRS regulations do not require companies to use CAPM when determining a discount rate and because economists have proposed other approaches to determine this figure, evaluating other methods is merited. One of the most well-known alternatives to CAPM is the Fama-French three-factor model.

The Fama-French model is named after its two developers, Nobel Prize-winning economist Eugene F. Fama and coauthor Kenneth R. French. It is not a completely different alternative to CAPM; rather, it builds on it. Fama-French uses CAPM concepts and adds two other factors the authors believe have historically driven equity returns. First, Fama and French believe smaller companies have historically earned higher returns than larger companies, and CAPM does not capture this. Second, they contend that companies with higher book value to market value ratios, or what are frequently called value stocks, have earned higher returns than CAPM would predict. The Fama-French model does not ignore CAPM. Instead, it begins with CAPM and adjusts the discount rate based on company size and a company's book-to-market-value ratio. Fama and French argue that their three-factor model measures the required return on equity investments more accurately than CAPM. The purpose of this report is to explain the Fama-French model and to assess whether that model is appropriate to determine a CSA's discount rate.

The Weighted Average Cost of Capital

A company's weighted average cost of capital (WACC) is frequently used as its discount rate. In *Advanced Corporate Finance*, Joseph Ogden, Frank C. Jen, and Philip O'Connor write: "A company's WACC can be interpreted as the implicit discount rate used by the market on the company's future cash flows to determine the value of the company's assets under a specified capital structure."[6] If we could rely on historical figures to calculate a company's WACC, calculating a discount rate would not be so challenging. But economists agree that the WACC needs to be a forward-looking figure, not a historical one. Investors make decisions based on expected future returns, not historical performance. Thus, companies must determine what it would cost today to raise new debt and equity to calculate their WACCs. Because interest rates change regularly, so do discount rates. Further, a company should use a forward-looking capital structure to calculate its WACC, not its historical mix of debt and stock. Eugene Brigham and Michael C. Ehrhardt write: "The WACC is used to find the present value of *future* cash flows, so it would be inconsistent to use weights based on the past history of the company."[7] (Emphasis in original.)

Further, a company's products and services, its competition, and its customer requirements are constantly changing, so the risks that a company faces in the future may be very different than in its past. Perhaps a company is shifting its strategic direction toward markets that have more business risk than the company has historically faced. Using a historical discount rate would be incorrect given a company's strategic direction, competition, and markets.

Treasury regulations cite the WACC as a figure that companies can use as a discount rate. The preamble to the temporary regulations concerning discount rates states: "For these reasons, an appropriate discount rate for the differential income stream might be determined based, for example, on the weighted average cost of capital of uncontrolled companies whose resources, capabilities, or rights are similar to the platform contributions and cost-shared intangibles under the CSA."[8]

Capital includes debt and equity instruments, such as bonds and stocks. It does not include current liabilities that do not pay interest, such as accounts and wages payable. In general, there are three sources of capital: interest-bearing debt, preferred stock, and common stock. A company's WACC should be determined by its mix of debt, preferred stock, and common stock, multiplied by

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[3]*Veritas Software Corp. v. Commissioner*, 133 T.C. 297 (2009).

[4]Shannon Pratt and Roger Grabowski, *Cost of Capital in Litigation* (2011).

[5]*See* David Stewart, "Lack of Guidance on Cost Sharing Regs Is Good, Official Says," *Tax Notes Int'l*, Aug. 1, 2011, p. 337.

[6]Ogden, Jen, and O'Connor, *Advanced Corporate Finance: Policies and Strategies* 33 (2003).

[7]Brigham and Ehrhardt, *Financial Management: Theory and Practice* 360 (14th ed. 2014).

[8]T.D. 9569, 76 F.R. 80250.

each source's cost. We can determine investor expectations by calculating a company's cost of raising capital. Brigham and Ehrhardt state: ''This rate of return from an investor's perspective is the cost from the company's point of view. Therefore the rate of return required by investors is called the Weighted Average Cost of Capital.''[9]

CAPM and the Fama-French model are both approaches to determine the required return on common stock, one component of the WACC. The cost of interest-bearing debt and preferred stocks can be calculated without CAPM or the Fama-French model. I begin with an explanation of the WACC and then discuss CAPM and how the Fama-French model can be used to determine the required return on equity.

Calculating the WACC

The standard formula to calculate a company's WACC is:

$$WACC = (W_{debt} (1 - t) (R_{debt})) + (W_{preferred} R_{preferred}) + (W_{common} R_{common})$$

In that formula:

R = required return for each cost of capital (debt, preferred stock, common stock)

W = weight of each component as percent of total capital

t = marginal corporate tax rate

Interest expenses are generally tax deductible, so the cost of debt should be calculated net of income taxes. However, preferred and common stock dividend payments are not tax deductible, so these should be pretax figures.

Suppose a company will be funded through 50 percent debt, 5 percent preferred stock, and 45 percent common stock. Further, suppose the company's marginal tax rate is 30 percent, the pretax cost of debt is 5 percent, preferred stock pays 6 percent, and the required return on common stock is 14 percent. Given this, the company's WACC would be:

$$WACC = ((0.5 (1 - 0.3) (0.05)) + (0.05) (0.06) + (0.45) (0.14)$$

$$WACC = 0.0175 + 0.003 + 0.063 = 0.0835 \text{ or } 8.35 \text{ percent}$$

As mentioned, the cost of capital should reflect current market rates. Robert F. Bruner et al. state:

> First, the capital costs appearing in the equation should be current costs reflecting current financial market conditions, not historical, sunk costs. In essence, the costs should equal the investors' anticipated internal rate of return on future cash flows, associated with each form of capital. Second, the weights appearing in the equation should be market weights, not historical weights based on often arbitrary, out-of-date book values.[10]

To determine the cost of long-term debt or preferred stock, a company could identify public offerings of comparable companies that have recently raised capital. Companies should locate businesses in the same industry, ideally of similar size and risk. This will admittedly be easier for large, publicly held companies, which may be able to find comparable companies more easily. Privately held companies may find it more difficult to identify similar businesses. Nonetheless, the costs that larger, publicly held companies pay to raise debt and preferred equity can establish useful benchmarks that smaller companies can use to determine their cost of debt and preferred stock.

While there are challenges in determining the cost of debt and preferred stock returns, the required return for common equity is the most challenging figure to calculate. As Bruner et al. write: ''As our survey results demonstrate, the most nettlesome component of WACC estimation is the cost of equity capital; for unlike readily observable yields in bond markets, no observable counterpart exists for equities.''[11] Because required returns on common stock are not observable, it is more difficult to determine that figure.

CAPM

CAPM has proved to be the most popular approach to calculate the required return on common stock, and it was used by both Veritas and the IRS in litigation.[12] It has been used in other court cases to value investments, so many legal precedents support CAPM. It is also the most commonly used approach in the industry. According to Bruner et al., 81 percent of companies, 80 percent of financial advisers, and 100 percent of textbooks used CAPM to determine the cost of equity.[13]

CAPM was developed by several financial economists in the 1960s, building on Harry Markowitz's modern portfolio theory. CAPM made several key contributions to our understanding of how investors can reduce risk and value securities. Shannon Pratt and Roger Grabowski write:

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[9]Brigham and Ehrhardt, *supra* note 7, at 12.

[10]Bruner et al., ''Best Practices in Estimating Cost of Capital: Survey and Syntheses,'' in *Financial Practice and Education* 13-28, at 14 (1998).

[11]*Id.*

[12]*Veritas*, 133 T.C. 297.

[13]*Id.* at 17.

> The CAPM is a conceptual cornerstone of modern capital market theory. Its relevance to business valuations and capital budgeting is that businesses, business interests, and business investments are a subset of the investment opportunities available to total capital markets; thus, the determination of the prices of businesses theoretically should be subject to the same economic forces and relationships that determine the prices of other investment assets.[14]

Those key forces include future cash flow, discount rates, and business risk. While CAPM has many implications, particularly for modern stock portfolio theory, my purpose is limited to determining a common stock's required rate of return in a CSA. However it should be noted that Richard Brealey, Stewart Myers, and Franklin Allen call CAPM a ''theory'' about the way markets should measure risk and return and that its advocates did not develop the model based on the historical performance of the stock market.[15]

The standard CAPM formula is:

$$E_r = R_f + \beta (R_m - R_f)$$

When:

E_r = the expected return on common stock

R_f = the risk-free rate of return

β = beta, or the correlation between this stock's return and the market's return

R_m = the market return on equity investments

The difference between the market rate of return for equities, R_m, and the risk-free rate, R_f, is often called the equity risk premium (ERP). This is the additional return that investors require to risk funds in equity markets.

In a previous article, I explained how a company structuring a CSA might determine the risk-free rate of return, the market return on equity investments, and the beta.[16] Briefly, investors could use the interest rate paid for a Treasury bill or bond as the risk-free rate of return. Bruner et al. say that approximately two-thirds of companies use the rate on Treasury bonds that are 10 years or longer for the risk-free rate.[17] Using a long-term Treasury bond rate is appropriate when determining the discount rate for a long-term investment, such as a CSA. A company could also use the long-run historical performance of the stock market for the market return on equity investments, using sources such as the *Ibbotson SBBI 2015 Classic Yearbook* for this figure.[18]

It is the third figure, the beta, that determines the required rate of return for a particular common stock. The beta is the correlation between a stock's rate of return and the market's return on all equity investments. If the beta is less than 1, the stock's value fluctuates less than the entire market. Because this equity is less risky than the market, the expected return on this stock is lower than it is for riskier securities. If its beta is greater than 1, a common stock is more volatile than the entire market, and thus, its required return should be higher than for a safer investment. A beta of exactly 1 means the stock moves up and down at the same rate as the market. In short, according to CAPM, beta measures risk.

For example, suppose the risk-free rate of return is 3 percent, the required return on all common stock investments is 9 percent, and the beta of a particular equity investment is 0.8. In this case, the required rate of return on this stock is:

$$R_{common} = 0.03 + 0.8(0.09 - 0.03)$$

$$R_{common} = 0.03 + 0.048 = 0.078 = 7.8 \text{ percent}$$

If the beta on that stock was higher, perhaps 1.5, investors would require a higher rate of return to compensate for risk. Applying the same formula:

$$R_{common} = 0.03 + 1.5(0.09 - 0.03)$$

$$R_{common} = 0.03 + 0.09 = 0.12 = 12 \text{ percent}$$

In short, investors expect a higher rate of return on stocks with high betas to compensate for the risk assumed. They accept a lower rate of return for less volatile investments.

This formula may give the incorrect impression that calculating a discount rate is straightforward. That is not the case. Economists may use different sources for the risk-free rate and equity market returns. For the risk-free rate, one might prefer to use a short-term Treasury bill rate, while another might believe long-term Treasury bonds should be used. There are also different sources for the long-run historical performance of the stock market, and these sources may calculate different figures. They may track different stock market indexes, begin their analyses at different dates, or account for survivorship bias in different ways. Thus financial economists may determine different risk-free and equity market returns.

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[14]Pratt and Grabowski, *supra* note 4, at 33.

[15]Brealey, Myers, and Allen, *Principles of Corporate Finance* 204 (11th ed. 2014).

[16]Webber, ''Determining the Discount Rate in a U.S. Cost-Sharing Agreement,'' *Tax Notes Int'l*, Sept. 9, 2013, p. 1013.

[17]Bruner, *supra* note 11, at 17.

[18]*Ibbotson SBBI 2015 Classic Yearbook: Market Results for Stocks, Bonds, Bills and Inflation, 1926-2014* (2015).

Nonetheless, calculating CAPM's beta poses the biggest challenge. To determine the correlation between the performances of a particular stock and the entire market, we have to rely on historical information. But the beta should be forward-looking. This is inherently speculative. If we rely on history, should we use one year of history, five years, or some other figure? Tracking history for a longer period may appear to make a beta more reliable, but if the company's risks have changed recently, the older information may no longer be pertinent. Should we correlate returns daily, weekly, monthly, or some other period? Not everyone agrees on the answer to that question. So even if economists use historical information, they are likely to calculate different betas and thus, different required rates of return. Bruner et al. reviewed betas for 22 companies published by Bloomberg, Value Line, and Standard and Poor's and found substantial differences in calculated betas for the same companies, with mean betas ranging from 1.02 to 1.24. In one instance, Standard and Poor's calculated a company's beta at 1.96, while Bloomberg computed 1.22.[19] Those are neutral sources with no financial incentives to arrive at higher or lower discount rates. Because of differences in international income tax rates, an MNE is likely to have financial motivations when determining a CSA's discount rate and thus the buy-in payment. Given the financial stakes involved, disputes with the IRS over discount rates seem likely.

Criticisms of CAPM

While CAPM is the most widely used approach to determine the required return on equity, it is not universally accepted. It is a theoretical model concerning the way many economists believe markets should assess risk and return. But it is not an empirical model based on the stock market's actual performance. In their widely used textbook, *Principles of Corporate Finance*, Brealey, Myers, and Allen write:

> But that does not mean that the capital asset pricing model is ultimate truth. We will see later that it has several unsatisfactory features, and we will look at some alternative theories. Nobody knows whether one of these alternative theories is eventually going to come out on top or whether there are other, better models of risk and return that have not yet seen the light of day.[20]

While CAPM is widely used and the centerpiece of many financial textbooks, many believe it is incomplete or flawed. Brealey, Myers, and Allen say the stock market's actual performance differs from what CAPM would have predicted. CAPM argues that high-beta stocks should earn higher rewards than low-beta securities. The historical evidence confirms that this is true. However, the difference between high- and low-beta stocks is not as large as CAPM would have predicted. Low-beta stocks have produced higher returns than CAPM predictions, and high-beta stocks fell below CAPM expectations. Brealey, Myers, and Allen write: "While high-beta stocks performed better than low-beta stocks, the difference was not as great as CAPM predicts."[21] There was a positive relationship between risk and reward, but the slope of the risk-reward line was flatter than CAPM expectations.

The evidence also uncovered several anomalies. One was that smaller stocks performed better than large stocks. Brealey, Myers, and Allen write: "You can see that small cap stocks did not always do well, but over the long haul their owners have made substantially higher returns. Since the end of 1926 the average annual difference between the returns on the two groups of stocks has been 3.6 percent."[22] Small stocks did not outperform larger securities every year, but over long periods they produced significantly higher returns.

The historical record also showed value stocks outperformed growth investments. Brealey, Myers, and Allen write: "Value stocks here are defined as those with high ratios of book value to market value. Growth stocks are those with low ratios of book to market. Notice that value stocks have provided a higher long-run return than growth stocks. Since 1926 the average annual difference between the returns on value and growth stocks has been 4.8 percent."[23] Again, value stocks did not outperform growth stocks every year, but over long periods their returns were significantly higher.

Brealey, Myers, and Allen acknowledge that this information challenges CAPM, but they are not ready to discard that model:

> There is no doubt that the evidence on the CAPM is less convincing than scholars once thought. But it will be hard to reject the CAPM beyond all reasonable doubt. Because the data and statistics are unlikely to give final answers, the plausibility of the CAPM *theory* will have to be weighed along with the empirical 'facts'.[24] (Emphasis in original.)

[19]Bruner et al., *supra* note 10, at 21.
[20]Brealey, Myers, and Allen, *supra* note 15, at 202.
[21]*Id.*
[22]*Id.* at 203.
[23]*Id.* at 203-204.
[24]*Id.* at 204.

Thus, we are left with an interesting problem. When determining a forward-looking required return on equity, should we rely on CAPM's theory of how markets should operate or on the actual performance of equity markets?

Similarly, Brigham and Ehrhardt acknowledge that legitimate questions have been raised about CAPM's accuracy, but they believe the model has merit, writing: ''Still, in our judgment, it is possible to develop 'reasonable' estimates of the required inputs, and we believe that the CAPM can be used to obtain reasonable estimates of the cost of equity. Indeed, despite the difficulties we have noted, surveys indicate that the CAPM is by far the most widely used method.''[25] It is the most popular model to determine the required return on equity, but it seems worthwhile to ask whether superior approaches have been developed. CAPM may indeed produce reasonable estimates, but the question is whether better estimates for the required return for equity can be calculated.

The Fama-French Three-Factor Model

CAPM assumes that the risk of an equity investment can be captured in one figure, the beta, which determines its required return. However for over 20 years academic studies have argued an investment's beta does not capture all investment risk and thus return. As mentioned, some studies have concluded that smaller companies, as measured by the market value of a company's equity, have outperformed larger companies. Other papers have concluded that value stocks have produced higher returns than growth investments. A commonly used metric for value stocks is the ratio of book value to market value of equity, sometimes abbreviated as the B/M ratio. These observations do not necessarily undermine CAPM, if these risks are captured by the beta. But several papers have concluded that the beta does not capture those two risks. Reviewing these results, Brigham and Ehrhardt write: ''This pattern alone would not be a challenge to CAPM if small firms and high B/M firms had large betas (and thus higher returns). However even after adjusting for their betas, the small stock portfolios and the high B/M portfolios earned higher returns than predicted by CAPM.''[26]

In 1993 Fama and French proposed adding the two factors to CAPM to improve projections of required equity returns.[27] Fama and French have elaborated and expanded on their ideas in many papers since, and their work has been deeply examined and discussed. My objective is to determine whether their model has sufficient credibility to be used when determining a CSA's discount rate and whether there are situations in which Fama-French could be the preferred alternative.

Fama and French analyzed long-run historical stock market performance and determined that both previously cited observations were correct. First, they determined that smaller companies, as measured by the market value of a company's equity, outperformed larger companies. Second, they found that value stocks, with high book-to-market ratios, outperformed growth investments. Brigham and Ehrhardt write: ''When Fama and French tested their hypotheses, they found that small companies and companies with high B/M ratios had higher rates of return than the average stock, just as they hypothesized.''[28] Further, they concluded that the CAPM model did not capture these returns. Thus, the CAPM beta did not measure an investment's entire risk. In contrast with CAPM, which described how economists thought stock markets should perform, Fama and French's analysis was based on the stock market's actual performance.

To address this, Fama and French proposed using three factors to estimate the required return for common stock investments. First, they used the CAPM ERP, which is the difference between the equity market's performance and the risk-free rate of return. Second, they proposed adding a premium for small companies. Third, they proposed adding a value stock premium. In short, this was not a complete replacement to CAPM. They retained the ERP but proposed adding small company and value stock premiums.

Brigham and Ehrhardt describe the Fama-French approach this way: ''The first factor in the Fama-French three-factor model is the market return, r_M, minus the risk-free rate, r_{RF}. Thus, their model begins like the CAPM, but they go on to add a second and a third factor.''[29] Describing their own model, Fama and French explain the need to retain the ERP:

> Because of its strong theoretical standing, the excess market return is one of the three risk factors in the model, and our tests confirm that it is important. It captures strong common

[25]Brigham and Ehrhardt, *supra* note 7, at 372.
[26]*Id.* at 270.
[27]Fama and French, ''Common Risk Factors in the Returns of Stocks and Bonds,'' 33 *J. Fin. Econ.* 3-56 (1993).
[28]Brigham and Ehrhardt, *supra* note 7 at 270-271.
[29]*Id.* at 271.

> time-series variation in returns, and the market premium is needed to explain the large differences between the average returns on stocks and bills.[30]

Concerning the size premium, Brigham and Ehrhardt describe its construction:

> To form the second factor, they ranked all actively traded stocks by size and then divided them into two portfolios, one consisting of small stocks and one consisting of big stocks. They calculated the return on each of these two portfolios, and created a third portfolio by subtracting the return on the big portfolio from that of the small one. They called this the SMB (small minus big) portfolio. This portfolio is designed to measure the variation in stock returns that is caused by the size effect.[31]

Brigham and Ehrhardt describe the value premium this way:

> To form the third factor, they ranked all stocks according to their book-to-market ratios (B/M). They placed the 30 percent of stocks with the highest ratios into a portfolio that they called the H portfolio (for high B/M ratios). They placed the 30 percent of stocks with the lowest ratios into the L portfolio (for low B/M) ratios. They subtracted the return of the L portfolio from the H portfolio, and they called the result the HML (high minus low) portfolio.[32]

This becomes the value stock premium.

Ogden, Jen, and O'Connor conclude that the Fama-French model does a very good job of capturing the actual performance of the stock market. Describing one of the Fama-French papers, they say:

> Using data for the years 1963 through 1994, they report the average monthly returns on portfolios of U.S. stocks cross-sorted into book-to-market equity ratio quintiles and firm size quintiles. . . . The results are indeed impressive. For each book-to-market quintile, average returns are generally inversely related to firm size, and for each size quintile, average returns are directly related to book-to-market equity ratio.[33]

In other words, the data support the small company and value stock premiums.

Pratt and Grabowski cite several advantages of the Fama-French model, emphasizing that it is based on historical data. They write:

> Because of the poor empirical record of pure CAPM, Eugene Fama and Kenneth French (FF) conducted an empirical study confirming that company size (as measured by market capitalization), earnings-to-price ratio, debt-to-equity ratio, and book-value-to-market-value ratios add to the explanation of realized returns provided by market beta. They found that the CAPM cost of equity estimates for high-beta stocks [was] too high and estimates of low-beta stocks were too low (relative to realized returns). The implication of their research is that if market betas do not suffice to explain expected returns, pure CAPM has potentially fatal problems. As a result, they introduced an empirically driven model to estimate cost of equity capital that is not dependent on beta alone.[34]

Further, Fama and French's model implicitly questions one of CAPM's fundamental assumptions. They write: "As such, FF considered that investors are not constrained to behave rationally, a tenet of pure CAPM."[35] In recent years, behavioral economists have challenged the assumption that investors act rationally, and they have argued that psychological factors, such as herding behavior, can result in mispriced assets, speculative bubbles, and collapses. The key CAPM assumption, that investors act rationally, is no longer universally accepted, which may be another advantage of the Fama-French model.

Fama-French Example

The formula for calculating Fama-French three-factor discount rates is shown below:

$$E(R_i) = R_f + \beta_i (R_m - R_f) + (s_i \times SMB) + (h_i \times HML)$$

Where:

$E(R_i)$ = expected rate of return on security

R_f = the risk-free rate of return

β = beta, or the correlation between this stock's return and the market's return

R_m = the market return on equity investments

s_i = small-minus-big coefficient in the Fama-French regression

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[30]Fama and French, "The CAPM is Wanted, Dead or Alive," 51 *J. Fin.* 1947, 1948 (1996).

[31]Brigham and Ehrhardt, *supra* note 7, at 271.

[32]*Id.*

[33]Ogden, Jen, and O'Connor, *supra* note 6, at 283.

[34]Pratt and Grabowski, *supra* note 4, at 42.

[35]*Id.*

SMB = expected small-minus-big risk premium, estimated as the difference between the historical average returns on the small-cap and large cap portfolios

h_i = high-minus-low coefficient in the Fama-French regression

HML = expected high-minus-low risk premium, estimated as the difference between the historical average returns on the high book-to-market and low book-to-market portfolios

The β in the Fama-French model is similar to, but not identical to, the β in CAPM. The CAPM beta is assumed to capture all of an investment's risk. However, in the three-factor model it captures only a portion of the risk. The size premium and the value premium also capture a share of the risk. From a mathematical perspective, Brealey, Myers, and Allen write: ''Note that we used *simple* regression to estimate β in the CAPM formula.''[36] But when describing the three-factor model's beta, they say it was calculated ''from a *multiple* regression of stock returns on the three factors.''[37] (Emphases in original.) In this way CAPM and Fama-French betas differ. Further, CAPM is arguably a simpler model than Fama-French, which may also contribute to CAPM's popularity. It is relatively easy to obtain a risk-free rate of return, the market's rate of return, and a security's CAPM beta. In contrast, calculating a Fama-French three-factor beta rate requires multiple regression analysis, which many will find more challenging.

To demonstrate this formula, suppose a company decided to calculate a three-factor discount rate in a CSA. We can again suppose that the risk-free rate of return is 3 percent and the market return on equity investments is 9 percent, so the ERP is 6 percent. I will suppose the three-factor beta is 0.8, again derived from a multiple regression, rather than the CAPM simple regression. In this example I will use the size and value premiums Brealey, Myers, and Allen have used. They wrote: ''Between 1926 and 2011 the difference between the annual returns on small and large capitalization stocks averaged 3.6 percent per year, while the difference between high and low book-to-market ratios averaged 4.8 percent.'' The 0.2 small-minus-big coefficient indicates this equity should capture only a small portion of the small stock premium. The 0.8 high-minus-low coefficient indicates it should receive a large portion of the value stock premium. Given these figures, the Fama-French discount rate would be:

Er = 3 percent + (0.8)(9 percent - 3 percent) + (0.2)(3.6 percent) + (0.8)(3.8 percent)

Er = 3 percent + 4.8 percent + 0.72 percent + 3.04 percent

Er = 11.56 percent

The figures above are meant to be only an example of the Fama-French calculations. A financial economist would want to get the latest information on the risk-free rate of return, the market rate of return, and the SMB and HML premiums. French maintains a data library with the latest financial information on his website.[38] This information on the French website can also be used to perform multiple regressions to calculate three-factor betas, small-minus-big coefficients, and the high-minus-low coefficients.

Ibbotson SBBI 2015 Classic Yearbook

Further support for the size and value premiums can be found in the *Ibbotson SBBI 2015 Classic Yearbook*, one of the most popular sources that companies and financial analysts use to determine the cost of capital. The yearbook dedicates one chapter to each premium, citing both favorably. In fact, its analysis of stock market returns reports separate figures for small-cap stocks and larger equities. It should also be noted that the SBBI yearbook uses data from the Center for Research in Security Prices (CRSP) from the University of Chicago's Booth School of Business, the same source Fama and French use for their research.

Concerning the size premium, the 2015 yearbook states: ''One of the most remarkable discoveries of modern finance is the finding of a relationship between company size and return. Historically, on average, small companies have higher returns than those of large ones.''[39] Further, the book states, ''the relationship between company size and return cuts across the entire size spectrum; it is not restricted to the smallest stocks.''[40] Since December 31, 1925, the SBBI yearbook says small stocks have grown at a 12.2 percent annual rate, while large cap stocks have grown at a 10.1 percent rate. As a result, $1 invested in the large-cap stocks on that date would be worth $5,316.85 by the end of 2014 and $27,419.32 if invested in small cap stocks.[41] This 2.1-point difference makes an enormous difference when compounded for approximately 90 years.

[36]Brealey, Myers, and Allen, *supra* note 15, at 208.
[37]*Id.*

[38]French's website is available at http://mba.tuck.dartmouth.edu/pages/faculty/ken.french. Go to the Data Library tab for the historical financial information, which extends back to 1926.
[39]*Ibbotson SBBI 2015 Classic Yearbook*, *supra* note 18, at 99.
[40]*Id.*
[41]*Id.* at 38.

The SBBI yearbook also concludes that CAPM's beta does not explain the higher returns of small securities. It states: ''The company size phenomenon is remarkable in several ways. First, the greater risk of small-cap does not, in the context of the capital asset pricing model, fully account for the higher returns over the long-term. In the CAPM, only systematic, or beta, risk, is rewarded; small-cap stock returns have exceeded those implied by their betas.''[42] In other words, CAPM's beta does not adequately capture small stock risk.

As mentioned previously, Fama and French calculated the small company premium by constructing three size portfolios and measuring performance differences. The CRSP database can be used to create size and value portfolios with more precision. In the SBBI yearbook the size portfolios are divided into 10 deciles, which are rebalanced four times a year using closing share prices at the end of each calendar quarter. At year-end 2014, the smallest two deciles (9 and 10) included companies with market capitalizations less than $549 million; deciles 6-8 were companies with market capitalizations between $549 million and $2.543 billion; deciles 3-5 included companies with market capitalizations between $2.552 billion and $10.106 billion, and companies with market capitalizations above $10.106 billion were in the largest two deciles. These figures change quarterly.

The *Ibbotson SBBI 2015 Classic Yearbook* also dedicates a chapter to the value premium. Using data from 1928 to 2014 from the CRSP database, the yearbook separates stocks into four groups, based on company size and whether the companies were value or growth stocks. One included small companies that were value stocks (small-value stocks), a second was small companies that were growth securities (small-growth), a third was large-value stocks, and the fourth was large-growth stocks. They determined that value stocks outperformed growth stocks, both for small and large companies. Between 1928 and 2014, small-value stocks grew at a 14.1 percent geometrical rate, while small-growth stocks grew at 9.4 percent. Over the same years, large-value stocks grew at 11.3 percent, while large-growth stocks grew at 9.1 percent.[43] In other words, even when the companies were of comparable size, value stocks appreciated more rapidly than growth stocks. The yearbook states: ''The top two performers during this time period were small-value and large-value stocks, followed by small-growth and large-growth stocks. Over the period from 1928 to 2014, small-value stocks outperformed all other stock series in the graph.''[44]

The SBBI yearbook states:

> Value significantly outperformed growth across the market capitalization spectrum. In the large-cap arena, the extra return of value over growth was the expense of increased risk, as the standard deviation of large-value was 27.4 percentage points versus 20 percentage points for large growth. In the small cap series, small value significantly outperformed small growth and did so with lower volatility (32.2 percentage points versus 32.3 percentage points).[45]

In short, the SBBI yearbook supports the value premium.

Critics of the Fama-French Model

Financial economists generally agree that the three-factor model explains historical stock market performance better than CAPM. But not everyone agrees the model can be used to predict predict returns successfully. Some view Fama-French as a historical anomaly and believe it lacks predictive power. In other words, even if a small stock or value stock premium existed in the past, it may not continue into the future. Perhaps it is more difficult for small companies or value stocks to remain unnoticed today. The widespread knowledge of the Fama-French model itself may motivate more investors to purchase small stocks and value securities today, driving down future returns.

Brigham and Ehrhardt express this view, writing: ''There is no question that the Fama-French three-factor model does a good job in explaining *actual* returns, but how well does it perform in explaining *required* returns? In other words, does the model define a relationship between risk and compensation for bearing risk?''[46] (Emphases in original.) One of their concerns is that Fama-French doesn't seem to explain why small stocks and value securities should command higher returns than larger companies and growth stocks. CAPM says companies that assume more risk merit higher returns, which makes logical sense. But why should small companies and value stocks earn higher returns? Brigham and Ehrhardt write: ''In summary, the Fama-French model is very useful in identifying the unexplained component of a stock's return. However, the model is less useful when it comes to estimating the

[42] *Id.* at 99.
[43] *Id.* at 119.
[44] *Id.*
[45] *Id.*
[46] Brigham and Ehrhardt, *supra* note 7, at 272.

required return on a stock because the model does not provide a well-accepted link between risk and required return.''[47]

Despite voicing those concerns, Brigham and Ehrhardt explain how Fama-French model supporters respond to these criticisms. They write:

> Advocates of the model suggest that size and B/M are related to risk. Small companies have less access to capital markets than do large companies, which subjects small companies to greater risk in the event of a credit crunch — such as the one that occurred during the global economic crisis that began in 2007. With greater risk, investors require a higher expected return to induce them to invest in small companies.[48]

Aswath Damodaran is a professor of finance at New York University who publishes an ERP analysis annually. He also questions whether the small company and value premiums will continue. He acknowledges the Fama-French model has done a better job of capturing historical performance than CAPM, but he is not certain this will continue. He writes:

> In summary, while the empirical evidence supports the notion that small cap stocks have earned higher returns after adjusting for beta risk than large cap stocks, it is not as conclusive, nor as clean as it was initially thought to be. The argument that there is, in fact, no small cap premium and that (what) we have observed over time is just an artifact of history cannot be rejected out of hand.[49]

As mentioned, the 2015 Ibbotson yearbook supports the size premium. It answers Fama-French detractors this way:

> Most criticisms of the use of the size premium do not address the underlying reason for its existence. Small-cap stocks are still considered riskier investments than large cap stocks. Investors require an additional reward in the form of additional return, to take on the added risk of an investment in small-cap stocks. It is unlikely that in the future investors will require no compensation for taking on this additional risk.[50]

Brigham and Ehrhardt say the value premium is an additional way to capture risk. They write:

> Similar arguments apply for companies with high B/M ratios. If a company's prospects are poor, the company will have a low market value, which causes a high B/M ratio. Lenders usually are reluctant to extend credit to a company with poor prospects, so an economic downturn can cause such a company to experience financial distress. In other words, a stock with a high B/M ratio might be exposed to the risk of financial distress, in which case investors would require a higher expected rate of return to induce them to invest such a stock.[51]

Ogden, Jen, and O'Connor also discuss these conflicts over the small stock and value premiums. They write: ''Critics argue that the Fama and French model is ad hoc, without theoretical foundations. In defense, the authors have argued that both the book-to-market factor and the company size factor are *risk proxies.* To see why this might be so, consider why some companies would have higher book-to-market ratios than others. The risk interpretation is that a company with a high book-to-market equity ratio (i.e., a relatively low market equity value) is likely to be a *distressed* firm. Such a company may have sustained losses recently and consequently has a substantial risk of bankruptcy (and perhaps higher leverage as well). Likewise, the typical small company is chronically in a much more precarious position regarding failure than the typical large firm.''[52] (Emphases in original.)

What can explain the superior performance of value stocks? The yearbook says readers of Benjamin Graham and David Dodd's *Security Analysis* might believe ''the outperformance of value stocks is due to the market coming to realize the full value of a company's securities that were once undervalued.''[53] Further, the yearbook says, ''several academic studies have shown that the market overreacts to good news. This would lead us to conclude that there is more room for value stocks (which are more likely to have reported bad news) to improve and outperform growth stocks, which already have high expectations built into them.''[54] Again, that is consistent with the theories of behavioral economists, who believe herding behavior can lead markets to misprice securities.

The financial community appears divided over whether the Fama-French model will do a better job of estimating future discount rates than CAPM. Nonetheless, even critics acknowledge it has done a better job of explaining past performance than pure

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[47]*Id.* at 273.
[48]*Id.* at 272-273.
[49]Damodaran, ''Equity Risk Premiums (ERP): Determinants, Estimation, Implications — The 2015 Edition,'' at 39 (Mar. 2015).
[50]*Ibbotson SBBI 2015 Classic Yearbook*, *supra* note 18, at 113.
[51]Brigham and Ehrhardt, *supra* note 7, at 272.
[52]Ogden, Jen, and O'Connor, *supra* note 6, at 283.
[53]*Ibbotson SBBI 2015 Classic Yearbook*, *supra* note 18, at 120.
[54]*Id.*

CAPM. Courts have been using historical CAPM discount rates to resolve legal cases for years. Given that Fama-French explains historical performance better than CAPM, I believe the case for using the three-factor model is quite strong, despite its critics.

Use of Fama-French in Other Court Cases

The Fama-French three-factor model has been used and cited favorably in at least one court case. While that wasn't a tax dispute, it involved the discount rate that should be used to determine an investment's present value. This precedent supports using the Fama-French model in other situations, such as CSAs.

Union Illinois v. Union Financial Group[55] focused on the valuation of a small, distressed bank sold in 2001. After the bank was sold, a family of minority shareholders argued that the bank's sale price was too low and requested additional compensation. The case centered on financial projections and the rate that should be used to discount future cash flows. The court ultimately sided with the defendant and determined that the sales price was fair.

In reaching its decision, the court used the Fama-French three-factor model to determine that the sales price was appropriate. Both the plaintiff and the defendant used CAPM to arrive at a discount rate, although the plaintiff calculated a 10.43 percent rate and the defendants used 16 percent. The judge tested specific valuation assumptions using figures he calculated with the Fama-French model. The judge wrote:

> In contrast to Mayer, however, I used the three-factor Fama and French CAPM cost of 13.53 percent. The advantage of using that formula is that it attempts to better account for some factors that explain equity return than does the original CAPM. These factors include the relationship of market returns to underlying book value, which is a proxy that, among other things, helps capture the risk associated with possible insolvency and other problems in highly leveraged companies. Although the Fama-French three factor CAPM model is not wholly accepted, neither is the original CAPM itself. By better factoring in the real risks of leverage, the Fama-French model captures useful data that contributes to a more reliable and real-world cost of capital.[56]

Thus, there might be situations in which Fama-French is not only a credible alternative to CAPM but the preferred approach.

[55]*Union Illinois v. Union Financial Group*, 847 A.2d 340 (Del. Ch. 2004).

[56]*Id.* at 362-363.

Conclusion

Calculating the required return on equity is an inherently speculative activity because we must rely on historical information to determine a forward-looking discount rate. With that caveat, I believe that there is sufficient support for using the Fama-French model when calculating a CSA's discount rate. The Fama-French model is cited favorably in many journal articles; it is identified as a credible CAPM alternative in several of the most widely used financial textbooks; and its developers are highly respected economists, with Fama garnering the Nobel Prize in economics in 2013. The *Ibbotson SBBI 2015 Classic Yearbook* also cites it favorably, dedicating one chapter to the small company premium and another to the value stock premium. It has been used in at least one court case, and the judge said that it may be better suited to capture the real-world risks facing small and distressed companies.

While CAPM is more widely used than the Fama-French model, even CAPM's supporters acknowledge that Fama-French explains historical performance better than CAPM. CAPM appears to underestimate required returns on low-beta securities and to overestimate required returns on high-beta equities. There is persuasive historical evidence that small companies and value stocks have outperformed large companies and growth stocks, even if adjusting for the CAPM beta. There is no certainty any approach to calculate discount rates will predict future returns accurately, so we must rely on historical evidence to make our best estimates. If we choose to rely on historical evidence, the Fama-French model clearly explains past performance better than CAPM, which is a more theoretical model.

CAPM advocates sometimes argue that the Fama-French model does not explain why small companies and value stocks should earn a higher rate of return than large companies and growth stocks, but I believe advocates of the three-factor model effectively answer those questions. They argue that small companies and value stocks are riskier investments and the CAPM beta does not capture this. The judge in *Union Illinois* thought Fama-French was a better approach to calculate a discount rate for a small, financially distressed company. I believe there is a credible case that the Fama-French model is a reasonable model to use for all CSA discount rates but it may be the preferred approach for small or financially distressed businesses. Given the financial importance of a discount rate, it makes sense for companies to consider using Fama-French in a CSA, particularly if they are small companies or value stocks.